

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

A Review of Image Fusion Methods

Hatte S.C^{*1}., Shingate V.S²

*1,2 Department of Electronics, K.B.P. College Of Engineering Satara, Shivaji University Kolhapur,

Maharashtra, India

vsshingate@yahoo.co.in

Abstract

Image fusion is the process of combining relevant information from two or more images into a single image. The resulting image will be more informative than any of the input images. There are many image fusion methods. This paper present a literature review on some of the image fusion techniques for image fusion and propose novel higher order singular value decomposition (HOSVD) based image fusion algorithm. Image fusion depends on local information of source images, the proposed algorithm picks out informative image patches of source images to constitute the fused image by processing the divided subtensors rather than the whole tensor. The sum of absolute values of the coefficients (SAVC) from HOSVD of subtensors is employed for activity-level measurement to evaluate the quality of the related image patch, and a novel sigmoid-function-like coefficient-combining scheme is applied to construct the fused result.

Keywords: Image fusion, Discrete cosine transform, Discrete wavelet transform, higher order singular value decomposition (HOSVD), sigmoid function.

Introduction

Image fusion is the combination of two or more images from same scene to form a new image by using a certain algorithm so that the resulting image will be more suitable for human and machine perception or further image processing tasks such as segmentation, feature extraction, and target recognition [1]-[5]. Image fusion find application in the area of navigation guidance, object detection and recognition, medical diagnosis, satellite imaging for remote sensing, rob vision, military and civilian surveillance, etc. Image fusion systems are widely used in surveillance and navigation applications, for both military and domestic purposes [5], for example, Due to the limited depth-of-focus of optical lenses in CCD devices, it is often not possible to get an image that contains all relevant objects "in focus". To achieve all objects "in focus", a fusion process is required so that we get resultant image with all objects in focus. Image fusion methods can be broadly classified into two groups - spatial domain fusion and transform domain fusion.

The fusion methods such as averaging, Brovey method, principal component analysis (PCA) and IHS based methods fall under spatial domain approaches. The disadvantage of spatial domain approaches is that they produce spatial distortion in the fused image. Spatial distortion can be very well handled by frequency domain approaches on image fusion.

Laplacian pyramid [1], , Discrete Wavelet Transform (DWT) [3], Discrete Cosine Transform (DCT) [4] etc., image fusion methods comes under transform domain. These methods show a better performance in spatial and spectral quality of the fused image compared to other spatial methods of fusion. These transform domain based methods merge the transform coefficients using the classical weighted average strategy or the choose-max strategy and then obtain the fused result through the inverse transformation of the combined coefficients. A novel HOSVD-based image fusion, constructed multiple input images as a tensor and can evaluate the quality of image patches using HOSVD of subtensors. Then, it employed a novel sigmoid-function-like coefficient - combining scheme to obtain the fused result [9]. A tensor is a multidimensional array. More formally, an N-way or Nth-order tensor is an element of the tensor product of N vector spaces, each of which has its own coordinate system. Tensor-based information processing methods are more suitable for representing high-dimensional data and extracting relevant information than vector- and matrix based methods and thus receives lots of attention [6]-[8]. As one of most efficient tensor decomposition

techniques, higher order singular value decomposition (HOSVD). Motivated by the salient ability of HOSVD to represent high-dimensional data and extract features, this paper proposes a novel HOSVD based image fusion algorithm [7]. It is worthwhile to highlight several aspects of the proposed transform domain-based approach here.

First, consider two or multiple source images of the same scene and are somewhat similar i.e. the same physical structures in the environment. Fusion algorithms are input dependent . If source images do not have same physical structure in environment then solution is to preprocess the source images. One of the important pre-processing steps for the fusion process is image registration. After getting source images with same structure as require by proposed algorithm, construct them in to a tensor and employs the HOSVD technique to extract their features simultaneously. This paper picks out image patches which contains maximum information of source images to constitute the fused image by processing the divided subtensors rather than the whole tensor.

A slice of the core tensor results from HOSVD of subtensors reflects the quality of the related image patch. Unlike the conventional activitylevel measurements, which apply the absolute value of a single coefficient to evaluate the corresponding pixel, paper employs the sum of absolute values of coefficients (SAVC) as the activity-level measurement of the related patch. To adapt to different activity-level measurements (approximate or substantially different), propose a novel and flexible sigmoid-function-like coefficient-combining scheme, which incorporates the usual choose-max scheme and the weighted average scheme [9]. The remainder of this paper is organized as follows. Section II, covers the pyramid decomposition based fusion, Section III presents the discrete wavelet transform based fusion. In Section IV discrete cosine transform based fusion is explained. In Section V the short theory of tensor and HOSVD discussed, In Section VI image fusion using HOSVD is explained and conclusion is given in Section VII.

Pyramid Decomposition Based Fusion

Pyramid Fusion Algorithm is a fusion method in the transform domain. Various Pyramid based fusion techniques are FSD Pyramid, Laplacian Pyramid, Ratio-of-low-pass Pyramid, Gradient Pyramid, Morphological Pyramid contrast can be used for the image fusion using different fusion rules. In pyramid approach, pyramid levels obtained from the down sampling of source images are fused at pixel level depending on fusion rules. The fused image is obtained by reconstructing the fused image pyramid. An image pyramid consists of a set of low pass or band pass copies of an image, each copy representing pattern information of a different scale [1].

The basic idea is to construct the pyramid transform of the fused image from the pyramid transforms of the source images and then the fused image is obtained by taking inverse pyramid transform. Typically, every pyramid transform consists of three major phases: Decomposition, Formation of the initial image for recomposition., Recomposition . Decomposition is the process where a pyramid is generated successively at each level of the fusion. Merging the input images is performed after the decomposition process. This resultant image matrix would act as the initial input to the recomposition process. The finally decimated input pair of images is worked upon either by averaging the two decimated input images, selecting the first decimated input image or selecting the second decimated input image. The recomposition is the process wherein, the resultant image is finally developed from the pyramids formed at each level of decomposition. Each of the pyramidal algorithms considered by us differ in the way the decomposition is performed. The Recomposition phase also differs accordingly [2].

Discrete Wavelet Transform Based Fusion

The discrete wavelets transform (DWT) allows the image decomposition in different kinds of coefficients preserving the image information. Such coefficients coming from different images can be appropriately combined to obtain new coefficients, so that the information in the original images is collected appropriately. Once the coefficients are merged, the final fused image is achieved through the inverse discrete wavelets transform (IDWT), where the information in the merged coefficients is also preserved [3].



Fig. 1. Block diagrams of generic fusion schemes where the input images have identical [3]

The DWT is applied to both source images and a decomposition of each original image is achieved. The different black boxes as shown in fig.1, associated to each decomposition level, are coefficient corresponding to the same image spatial representation in each original image, i.e. the same pixel or pixels positions in the original images. Only coefficients of the same level and representation are to be fused, so that the fused multiscale coefficients can be obtained. This is displayed in the diagonal details where the curved arrows indicate that both coefficients are merged to obtain the new fused multiscale coefficient. This is applicable to the remainder coefficient. Once the fused multiscale is obtained, through the IDWT, the final fused image is achieved [3].

Discrete Cosine Transform Based Fusion

This paper studies image fusion in the DCT domain, also, present an image fusion technique based on a contrast measure defined in the DCT domain in JPEG framework. This technic is faster than the wavelet based image fusion technique when the images to be fused were saved in JPEG format or when the fused image will be saved or transmitted in JPEG format. There is no difference in visual quality between the fused image obtained by image fusion technique based on a contrast measure defined in the DCT domain and that obtained by the wavelet transform based image fusion technique.

In this algorithm first, divides up the original images into 8 by 8 pixel blocks, and then calculates the discrete cosine transform (DCT) of each block. A quantizer rounds off the DCT coefficients according to the quantization matrix. Quantization of the DCT coefficients is a lossy process. Then entropy coding is used to encode the quantized coefficients and a compression data stream is output. In the decoder, JPEG recovers the quantized DCT coefficients from the compressed data stream, takes the inverse DCT transform and displays the image. Here we use different fusion techniques to obtain the fused images

The key step is to fuse the DCT representations of multi-images into a single DCT representation of the fused image. A contrast sensitivity method is adopted to produce a visually better fused image. This is based on the fact that the human visual system is sensitive to local contrast [4].

Tensor and HOSVD

A tensor is a multidimensional or N-way array. Tensors (multiway arrays) are generalizations of scalars, vectors, and matrices to an arbitrary number of indices. A first-order tensor is a vector, a second-order tensor is a matrix, and tensors of order three or higher are called higher-order tensors [6].

In many areas of science and technology, data structures have more than two dimensions, and are naturally represented by multidimensional arrays or tensors. Two-dimensional matrix methods, such as the singular value decomposition (SVD), are widespread and well studied mathematically. However, they do not take into account the multidimensionality of data. Decomposition is a multilinear generalization of the SVD to multidimensional data. As the decomposition can have higher dimensions than 3, they called it higher order SVD (HOSVD). Decomposition applied to data arrays for extracting and explaining their properties [7]. Here, we introduce several notations and operations of tensors, which will be used in the rest of this paper (see [8] for details).

- 1. An Nth order tensor is an object with N indices, i.e., $A \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$.
- 2. An n^{th –} mode vector of an $(I_1 \times I_1 \times ... \times I_N)$ dimensional tensor A is an In- dimensional vector obtained by varying index in but fixing the indices.
- 3. The n^{th} mode product of a tensor $A \in \mathbb{R}^{l_1 \times l_2 \times \dots \times l_N}$ and a matrix $U \in \mathbb{R}^{l_n \times l_n}$ along the nth mode is denoted by

 $\mathbf{B} = U \in R^{I_1 \times I_2 \times \dots \times I_{n-1} \times J_n \times I_{n+1} \times \dots \times I_N}$ with elements $b_{i_1,i_2 \times \dots \times i_{n-1},j_n,i_{n+1} \times \dots \times i_N} =$

 $\sum_{i_n=1}^{l_n} a_{i_1,i_2 \times \dots \times i_{n-1},j_n,i_{n+1} \times \dots \times i_N} \bullet u_{j_n,i_n},$ where u_{j_n,i_n} stands for the $(j_n,i_n)^{\text{th}}$ element of matrix U, and $a_{i_1,i_2 \times \dots \times i_{n-1},j_n,i_{n+1} \times \dots \times i_N}$ represents the $(i_1, i_2 \times ... \times i_{n-1})$

- $i_n, i_{n+1} \times \dots \times i_N$) the lement of tensor A. The n^{th-}mode matricization of a tensor A is an 4. operation where the nth – mode vectors of A are aligned as the columns of a matrix which is denoted by A_(u).
- 5. HOSVD of tensor $A \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$ is given by $A = \sum_{i_1 \times I_2} \times_1 U_1 \times_2 U_2 \dots \times_N U_N$, where $\sum \in R^{I_1 \times I_2 \times \dots \times I_N}$ is the core tensor that satisfies the all-orthogonality conditions, and $U_n \in R^{J_n \times I_n}$, $n = 1, 2, \dots, N$, are the left singular vectors of $A_{(u)}$.

HOSVD Based Fusion

A. Description of Proposed Algorithm

Generally, a transform-domain fusion algorithm consists of the following three steps: 1) obtain the decomposition coefficients using some transform; 2) construct the activity-level measurement from these coefficients; and 3) merge these coefficients to construct the fused result in line with the measurements above. In the remainder of

this section, a new image fusion algorithm is developed according to the steps above. HOSVD is one of most efficient data-driven decomposition techniques and can extract the features of multiple slices of the decomposed tensor simultaneously. To facilitate the description, we begin with two $(M \times N)$ - dimensional gray images [9].

Step 1: Two source images are constructed into a tensor with $(M \times N \times 2)$ - dimensions (i.e., with three modes: the row, the column, and the label of the source image order), and HOSVD is employed to extract the related features (i.e., to obtain the decomposition coefficients). Although HOSVD is used to obtain the decomposition coefficients (or extract features) of multiple images, there are two important differences. we form $(\breve{M} \times \breve{N} \times 2)$ dimensional subtensors A_i, using two image patches $B_i(1)$ and $B_i(2)$ separately from the two source images and perform the HOSVD of A_i , so that informative image patches are picked out to piece together the final fused image. Use the nth-mode product of the core tensor and the third-mode factor matrix to reflect the quality of the related image patch for the purpose of constructing the final fused result from the product above more conveniently [8].

Step 2: It is commonly thought that the magnitude (absolute value) of the decomposed coefficient is consistent with the related local energy, which implies that the larger the absolute value of the coefficient is. the more information the corresponding pixel contains. Therefore, many transform domain fusion methods employ the absolute value of the coefficient as the activity-level measurement of the corresponding pixel. Borrowing the idea but unlike it, defines the SAVC as the activity-level measurement of the related image patch to evaluate its quality [9].

Step 3: To derive the coefficient-combining scheme, we first consider all possibilities: 1) In the same subtensor, the image patch with an even higher SAVC value contains more rich information or is of higher quality; thus, it should be directly selected as the final fused result of the corresponding subtensor (i.e., in this case, the choose-max strategy should be applied). 2) If the SAVCs of both image patches are close to each other, then they have approximate image quality, and thus, their weighted average should be used as the ultimate fused result of the subtensor (i.e., in this case, the weighted average strategy should be employed).

ISSN: 2277-9655 Impact Factor: 1.852



Fig. 2. Sigmoid function with different shrink factors [9]

However, for the first case, when two adjacent image patches are chosen, respectively, from different source images, it will cause discontinuous gap pixels between adjacent patches of the fused image. Therefore, a choose-max strategy with the smoothing function should be designed. In order to attain the aim above, this paper designs a novel sigmoid-function-like coefficient-combining scheme to adapt to different cases: the first situation is mapped into the flat region [marked in Fig.2] of the sigmoid function's range, and then, the approximate choose-max scheme works. The second one is projected into its steep region [marked in Fig.2], and in this case, the weighted average scheme works [9]. Thus, the proposed algorithm is summarized here.



1) Initialization

Construct two $(M \times N)$ -dimensional source images into a tensor with $(M \times N \times 2)$ -dimensions. To further avoid the discontinuous gap above, the consecutive subtensors are enabled to partly share data, i.e., a sliding window technique is applied here to divide the tensor into $(\tilde{M} \times \tilde{N} \times 2)$ -dimensional subtensors with moving step size p, which satisfies with $p \leq \tilde{M}$ and $p \leq \tilde{N}$. Note that I= fix $((M - \tilde{M} + 1) / p) \cdot fix ((N - \tilde{M} + 1) / p)$, where $((M - \tilde{M} + 1) / p)$ stands for the nearest integers (toward zero) of $((M - \tilde{M} + 1) / p)$. 2) For $i = 1, 2, \dots, I$, let the HOSVD of divided subtensor A_i be given by

$$A_i = \Sigma_i \times_1 U_i \times_2 V_i \times_3 W_i$$

To construct the fused result conveniently, we employ the following tensor:

$$\overline{\Sigma}_{i} = \Sigma_{i} \times_{3} W_{i}$$
(2)

as the features of image patches rather than the original core tensor Σ_i Based on $\overline{\Sigma}_i$, each image patch $B_i(l), l = 1, 2, \dots, \tilde{N}$, of subtensor A_i can be represented as

$$\mathbf{B}_{i}(\mathbf{l}) = \mathbf{U}_{i} \times \overline{\Sigma}_{i} (:, :. \mathbf{l}) \bullet \mathbf{V}_{i}^{T} , \ \mathbf{l} = 1, 2.$$
(3)

To facilitate the description, $\mathbf{B}_i(l)$ is lexicographically ordered as $vec(\mathbf{B}_i(l))$ in a vector form, i.e.,

$$vec(\mathbf{B}_{i}(\mathbf{l})) = \sum_{m=1}^{M} \sum_{n=1}^{\tilde{N}} \overline{\Sigma}i(m, n, \mathbf{l}) \bullet$$

 $vec(\mathbf{um} \times v_i^T)$ (4)

where \mathbf{u}_{m} represents the mth column of \mathbf{U}_{i} and \mathbf{v}_{n} stands for the nth column of \mathbf{V}_{i} . Since both \mathbf{U}_{i} and \mathbf{V}_{i} are orthogonal matrices, $vec(\mathbf{u}_{m} \times v_{i}^{T})$, m = 1,2,.... \breve{M} , n = 1,2, ,.... \breve{N} , form an orthogonal basis, which implies that element $\overline{\Sigma}_{i}$ (m, n, 1) is actually the projection coefficient of $vec(\mathbf{B}_{i}(1))$ on $vec(\mathbf{u}_{m} \times v_{i}^{T})$

Based on coefficient matrix $\overline{\Sigma}_i$ (:, :. 1) the activitylevel measurement of image patch $B_i(1)$ is defined as

$$e_{i}(l) = \sum_{m=1}^{M} \sum_{n=1}^{N} |\overline{\Sigma}i(m, n, l)|, \ l = 1,2$$

(5)

which can be represented in another form as

 $e_i(l) = \|vec(\overline{\Sigma}_i(:,:,l))\|_l, \quad l = 1,2$ (6) i.e., the activity-level measurement is the p1 norm of vector $vec(\overline{\Sigma}_i(:,:,l))$.

According to these activity-level measurements $e_i(l)$, l = 1, 2, coefficient matrices $\overline{\Sigma}_i(:,:1)$ and $\overline{\Sigma}_i(:,:2)$ are merged to obtain new coefficient matrix D_i , i.e.,

$$D_{i} = \frac{1}{1 + \exp\left(-k \ln\left(\frac{e_{i}(1)}{e_{i}(2)}\right)\right)} \times \overline{\Sigma}_{i}(:, :, 1) +$$

$$\frac{\exp\left(-k\ln\left(\frac{\mathbf{e}_{i}(1)}{\mathbf{e}_{i}(2)}\right)\right)}{1+\exp\left(-k\ln\left(\frac{\mathbf{e}_{i}(1)}{\mathbf{e}_{i}(2)}\right)\right)} \times \overline{\Sigma}_{i}(:,:2)$$
(7)

Where k is the shrink factor of the sigmoid function. After D_i is obtained, fused image patch F_i , is determined as follows:

 $F_i = U_i \times D_i \times V_i^T$, I = 1, 2,I (8) 3) Finally, fused imageis constructed with, F_i , i = 1, 2,I: a) Initialize G as a zero matrix, i.e., $G = 0_{M \times N}$; b) superimposed F_i onto G at its corresponding patch position, i = 1, 2,I; and c) for each pixel position of G, the added pixel value divided by its adding times is employed as the final fused result of this position.

B. Discussion of the Sigmoid Function First, we consider one of two limit cases, i.e. $k = +\infty$, .If $(e_i(1) / e_i(2)) > 1$, then

$$\frac{1}{1 + \exp\left(-k \ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} = 1$$
$$\frac{\exp\left(-k \ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)}{1 + \exp\left(-k \ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} = 0$$

Otherwise, If $(e_i(1) / e_i(2)) < 1$ if, then

$$\frac{1}{1 + \exp\left(-k\ln\left(\frac{e_{i}(1)}{e_{i}(2)}\right)\right)} = 0$$
$$\frac{\exp\left(-k\ln\left(\frac{e_{i}(1)}{e_{i}(2)}\right)\right)}{1 + \exp\left(-k\ln\left(\frac{e_{i}(1)}{e_{i}(2)}\right)\right)} = 1$$

Obviously, the proposed coefficient-combining strategy reduces into the choose-max strategy. In other words, the choose-max scheme is just the special case of the proposed coefficient-combining strategy, i.e.,

$$\begin{split} F_{i} &= U_{i} \times D_{i} \times V_{i}^{T} \\ &= \begin{cases} U_{i} \times \overline{\Sigma}_{i}(:,:1) \times V_{i}^{T} & \text{if } ei(1) > ei(2) \\ U_{i} \times \left(\frac{1}{2}\overline{\Sigma}_{i}(:,:1) + \frac{1}{2}\overline{\Sigma}_{i}(:,:2)\right) \times V_{i}^{T} \\ \text{otherwise} \\ U_{i} \times \overline{\Sigma}_{i}(:,:1) \times V_{i}^{T} & \text{if } ei(2) > ei(1) \end{cases} \\ &= \begin{cases} B_{i}(1) & \text{if } ei(1) > ei(2) \\ \frac{1}{2}(B_{i}(1) + B_{i}(2)) & \text{otherwise} \end{cases} (9) \end{split}$$

 $\begin{array}{c} B_i(2) & \text{if } ei(2) > ei(1) \\ \text{(Then, we consider another limit case, i.e., } k = 0. \\ \text{In this case} \end{array}$

$$\frac{1}{1 + \exp\left(-k\ln\left(\frac{e_{i}(1)}{e_{i}(2)}\right)\right)} = \frac{1}{2}$$
$$\frac{\exp\left(-k\ln\left(\frac{e_{i}(1)}{e_{i}(2)}\right)\right)}{1 + \exp\left(-k\ln\left(\frac{e_{i}(1)}{e_{i}(2)}\right)\right)} = \frac{1}{2}$$

Thus, the proposed algorithm is reduced into the average fusion method, i.e.

$$F_{i} = U_{i} \times D_{i} \times V_{i}^{T}$$

$$= U_{i} \times \left(\frac{1}{2}\overline{\Sigma}_{i}(:,:.1) + \frac{1}{2}\overline{\Sigma}_{i}(:,:.2)\right) \times V_{i}^{T}$$

$$= \frac{1}{2}(B_{i}(1) + B_{i}(2)) \qquad (10)$$

To facilitate the analyses, we plot the sigmoid function

$$\frac{1}{1 + \exp\left(-k \ln\left(\frac{e(1)}{e(2)}\right)\right)}$$

with different in Figure.4.1, From this figure, several aspects can be observed

1) As k increases

$$\frac{1}{1 + \exp\left(-k \ln\left(\frac{e(1)}{e(2)}\right)\right)}$$
approaches
$$\frac{1}{2} + \frac{1}{2} \operatorname{sgn}\left(\frac{e(1)}{e(2)} - 1\right)$$

Where sgn (•) is the sign function. In particular, when $k = +\infty$

$$\frac{1}{1 + \exp\left(-k \ln\left(\frac{e(1)}{e(2)}\right)\right)}$$

is equivalent to

 $\frac{1}{2} + \frac{1}{2} \operatorname{sgn}\left(\frac{e(1)}{e(2)} - 1\right)$

i.e., the coefficient-combining strategy proposed in reduces into the choose-max scheme.

- 2) When k = 0, the proposed algorithm is equivalent to the average fusion method.
- 3) For the same (e(1) / e(2)): If larger k is applied, the coefficient combining strategy plays the selection role. However, if smaller k is applied, the coefficient-combining strategy plays the average or smoothing function.
- 4) The same k : When e(1) is even larger or smaller thane (2), the coefficient- combining scheme plays the selection role. However, when e(1) is closer to e(2), the coefficientcombining strategy plays the weighted average role.

Conclusion

This review paper studies the different image fusion methods and proposes a novel HOSVD based image fusion algorithm. The success of the proposed algorithm lies in the following: 1) HOSVD, a fully data-driven technique, is an efficient tool for highdimensional data decomposition and feature extraction; 2) the SAVC is a feasible activity-level measurement for evaluating the quality of image the sigmoid-function-based patches; and 3) coefficient-combining strategy incorporates the conventional choose-max strategy and the weighted average strategy and thus adapts to different activity levels.

References

- [1] E. H. Adelson, C. H. Anderson, J. R. Bergen, P. J. Burt, and J. Qgden, "Pyramid methods in signal processing," RCA Eng., vol. 29, no. 6, pp. 33-41, Nov./Dec. 1984.
- [2] S. S. Bedi, Rati Khandelwal, " Comprehensive Comparative Study of Image Fusion and Techniques", IJSCE, Volume-3, Issue-1, March 2013,pp.300-304.
- [3] Gonzalo Pajares, Jes'us Manuel de la Cruz, "A wavelet-based image fusion tutorial ", Pattern Recognition, Vol.37 (2004) pp.1855 - 1872.
- [4] J. Tang, "A contrast based image fusion technique in the DCT domain," Digit. Signal Process., vol. 14, no. 3, pp. 218-226, May 2004.
- [5] Pohl & J. L. Van Genderen, "Review article Multisensor image fusion in remote sensing: Concepts, methods and applications", int. j. remote sensing, 1998, vol. 19, no. 5, pp. 823-854.
- [6] T. G. Kolda and B. W. Bader, "Tensor decompositions

and applications," SIAM Rev., vol. 51, no. 3, pp. 455-500, Sep. 2009.

- [7] G. Bergqvist and E. G. Larsson, "The higherorder singular value decomposition: Theory and application," IEEE Signal Process. Mag., vol. 27, no. 3, pp. 151-154, May 2010.
- [8] L. De Lathauwer, B. De Moor, and J. Vandewalle, "A multilinear singular value decomposition," SIAM J. Matrix Anal. Appl., vol. 21, no.4, pp. 1253-1278. Mar. -May 2000.
- [9] Junli Liang, Yang He, Ding Liu, and Xianju Zeng, "Image Fusion Using Higher Order Singular Value Decomposition" IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 21, NO. 5, MAY 2012, pp. 2898-2909.